CONSTRAINING THE SYMMETRY ENERGY FROM THE NEUTRON SKIN THICKNESS OF TIN ISOTOPES

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We show in the Skyrme-Hartree-Fock approach that unambiguous correlations exist between observables of finite nuclei and nuclear matter properties. Using this correlation analysis to existing data on the neutron skin thickness of Sn isotopes, we find important constraints on the value $E_{\rm sym}(\rho_0)$ and density slope L of the nuclear symmetry energy at saturation density. Combining these constraints with those from recent analyses of isospin diffusion and double neutron/proton ratio in heavy ion collisions leads to a value of $L = 58 \pm 18$ MeV approximately independent of $E_{\rm sym}(\rho_0)$.

Keywords: Nuclear symmetry energy; Neutron skin; Skyrme Hartree-Fock.

1. Introduction

The nuclear symmetry energy $E_{\rm sym}(\rho)$ plays a crucial role in both nuclear physics and astrophysics.[1,2] Although significant progress has been made in recent years in determining the density dependence of $E_{\rm sym}(\rho)$,[2] large uncertainties still exist even around the normal density ρ_0,[3] and this has hindered us from understanding more precisely many important properties of neutron stars.[4] To constrain the symmetry energy with higher accuracy is thus of crucial importance.

Theoretically, it has been established[5–13] that the neutron skin thickness $\Delta r_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$ of heavy nuclei, given by the difference of their neutron and proton root-mean-squared radii, provides a good probe of $E_{\rm sym}(\rho)$. In particular, Δr_{np} has been found to correlate strongly with both $E_{\rm sym}(\rho_0)$ and L in microscopic mean-field calculations.[5–11] It is, however, difficult to extract an accurate value for L from comparing the calculated Δr_{np} of heavy nuclei with experimental data as it depends on several nuclear interaction parameters in a highly correlated manner[7,8] and the calculations have been usually carried out by varying the interaction parameters. A well-known example is the Skyrme-Hartree-Fock (SHF) approach using normally 9 interaction parameters and there are more than 120 sets of Skyrme interaction parameters in the literature.

In the present talk, we report our recent work[14] on a new method to analyze the correlation between observables of finite nuclei and some macroscopic properties of asymmetric nuclear matter. Instead of varying directly the 9 interaction parameters within the SHF, we express them explicitly in terms of 9 macroscopic observables that are either experimentally well constrained or empirically well known. Then, by varying individually these macroscopic observables within their known ranges, we can examine more transparently the correlation of Δr_{np} with each individual observable. In particular, we have demonstrated that important constraints on $E_{\rm sym}(\rho_0)$ and L can be obtained with the application of this correlation analysis to existing data on the neutron skin thickness of Sn isotopes.

2. The theoretical method

In the standard SHF model,[15] the 9 Skyrme interaction parameters, i.e., σ, $t_0 - t_3$, $x_0 - x_3$ can be expressed analytically in terms of 9 macroscopic quantities ρ_0, $E_0(\rho_0)$, the incompressibility K_0, the isoscalar effective mass $m^*_{s,0}$, the isovector effective mass $m^*_{v,0}$, $E_{\rm sym}(\rho_0)$, L, the gradient coefficient G_S, and the symmetry-gradient coefficient G_V,[14,16] i.e.,

$$\begin{aligned}
&\sigma = \gamma - 1, \quad t_0 = 4\alpha/(3\rho_0), \quad x_0 = 3(y-1)E^{\rm loc}_{\rm sym}(\rho_0)/\alpha - 1/2, \\
&t_1 = 20C/\left[9\rho_0(k_{\rm F}^0)^2\right] + 8G_S/3, \quad x_1 = \left[12G_V - 4G_S - \frac{6D}{\rho_0(k_{\rm F}^0)^2}\right]/(3t_1), \\
&t_2 = \frac{4(25C - 18D)}{9\rho_0(k_{\rm F}^0)^2} - \frac{8(G_S + 2G_V)}{3}, \\
&x_2 = \left[20G_V + 4G_S - \frac{5(16C - 18D)}{3\rho_0(k_{\rm F}^0)^2}\right]/(3t_2), \\
&t_3 = 16\beta/\left[{\rho_0}^{\gamma}(\gamma+1)\right], \quad x_3 = -3y(\gamma+1)E^{\rm loc}_{\rm sym}(\rho_0)/(2\beta) - 1/2, \qquad (1)
\end{aligned}$$

where $k_{\rm F}^0 = (1.5\pi^2\rho_0)^{1/3}$, $E_{\rm sym}^{\rm loc}(\rho_0) = E_{\rm sym}(\rho_0) - E_{\rm sym}^{\rm kin}(\rho_0) - D$, and the parameters C, D, α, β, γ, and y are defined as[17]

$$
\begin{aligned}
C &= \frac{m - m_{s,0}^*}{m_{s,0}^*}E_{\rm kin}^0, \quad D = \frac{5}{9}E_{\rm kin}^0\left(4\frac{m}{m_{s,0}^*} - 3\frac{m}{m_{v,0}^*} - 1\right), \\
\alpha &= -\frac{4}{3}E_{\rm kin}^0 - \frac{10}{3}C - \frac{2}{3}[E_{\rm kin}^0 - 3E_0(\rho_0) - 2C] \\
&\quad \times \frac{K_0 + 2E_{\rm kin}^0 - 10C}{K_0 + 9E_0(\rho_0) - E_{\rm kin}^0 - 4C}, \\
\beta &= \left[\frac{E_{\rm kin}^0}{3} - E_0(\rho_0) - \frac{2}{3}C\right]\frac{K_0 - 9E_0(\rho_0) + 5E_{\rm kin}^0 - 16C}{K_0 + 9E_0(\rho_0) - E_{\rm kin}^0 - 4C}, \\
\gamma &= \frac{K_0 + 2E_{\rm kin}^0 - 10C}{3E_{\rm kin}^0 - 9E_0(\rho_0) - 6C}, \quad y = \frac{L - 3E_{\rm sym}(\rho_0) + E_{\rm sym}^{\rm kin}(\rho_0) - 2D}{3(\gamma - 1)E_{\rm sym}^{\rm loc}(\rho_0)}, \quad (2)
\end{aligned}
$$

with $E_{\rm kin}^0 = \frac{3\hbar^2}{10m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho_0^{2/3}$ and $E_{\rm sym}^{\rm kin}(\rho_0) = \frac{\hbar^2}{6m}\left(\frac{3\pi^2}{2}\rho_0\right)^{2/3}$.

As a reference for the correlation analyses below, we use the MSL0 parameter set,[14] which is obtained by using the following empirical values for the macroscopic quantities: $\rho_0 = 0.16$ fm^{-3}, $E_0(\rho_0) = -16$ MeV, $K_0 = 230$ MeV, $m_{s,0}^* = 0.8m$, $m_{v,0}^* = 0.7m$, $E_{\rm sym}(\rho_0) = 30$ MeV, and $L = 60$ MeV, $G_V = 5$ MeV·fm^5, and $G_S = 132$ MeV·fm^5. And the spin-orbital coupling constant $W_0 = 133.3$ MeV ·fm^5 is used to fit the neutron $p_{1/2} - p_{3/2}$ splitting in ^{16}O. Using other Skyrme interactions obtained from fitting measured binding energies and charge rms radii of finite nuclei does not change our conclusion.

3. Results

To reveal clearly the dependence of Δr_{np} on each macroscopic quantity, we vary one quantity at a time while keeping all others at their default values in MSL0. Shown in Fig. 1 are the values of Δr_{np} for ^{208}Pb, ^{120}Sn and ^{48}Ca. Within the uncertain ranges for the macroscopic quantities considered here, the Δr_{np} of ^{208}Pb and ^{120}Sn exhibits a very strong correlation with L. However, it depends only moderately on $E_{\rm sym}(\rho_0)$ and weakly on $m_{s,0}^*$. On the other hand, the Δr_{np} of ^{48}Ca displays a much weaker dependence on both L and $E_{\rm sym}(\rho_0)$. Instead, it depends moderately on G_V and W_0. This explains the weaker Δr_{np}-$E_{\rm sym}(\rho)$ correlation observed for ^{48}Ca in previous SHF calculations using different interaction parameters.[10]

Experimentally, the Δr_{np} of heavy Sn isotopes has been systematically measured.[18–23] As an illustration, we first show in the panel (a) of left

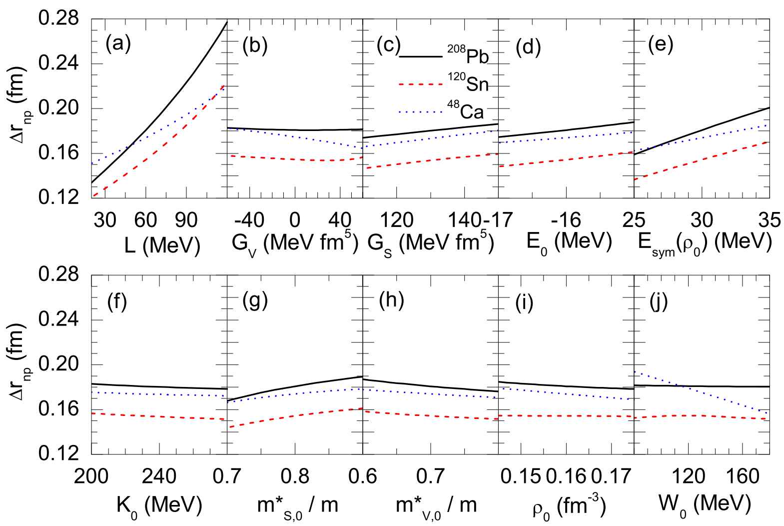

Fig. 1. (Color online) The neutron skin thickness Δr_{np} of ^{208}Pb, ^{120}Sn and ^{48}Ca from SHF with MSL0 by varying individually L (a), G_V (b), G_S (c), $E_0(\rho_0)$ (d), $E_{\rm sym}(\rho_0)$ (e), K_0 (f), $m^*_{s,0}$ (g), $m^*_{v,0}$ (h), ρ_0 (i), and W_0 (j). Taken from Ref.[14]

window in Fig. 2 the comparison of the available Sn Δr_{np} data with our calculated results using 20, 60 and 100 MeV, respectively, for the value of L and the default values for all other quantities in MSL0. It is seen that the value $L = 60$ MeV best describes the data. To be more precise, the χ^2 evaluated from the difference between the theoretical and experimental Δr_{np} values is shown as a function of L in the panel (b) of left window in Fig. 2. The most reliable value of L is found to be $L = 54 \pm 13$ MeV within a 2σ uncertainty.

Since the value of Δr_{np} depends on both L and $E_{\rm sym}(\rho_0)$, we have carried out a two-dimensional χ^2 analysis as shown by the grey band in the panel (c) of left window in Fig. 2. It is seen that increasing the value of $E_{\rm sym}(\rho_0)$ systematically leads to smaller values of L. Furthermore, we have estimated the effects of nucleon effective mass by using $m^*_{s,0} = 0.7m$ and $m^*_{v,0} = 0.6m$ as well as $m^*_{s,0} = 0.9m$ and $m^*_{v,0} = 0.8m$, in accord with the empirical constraint $m^*_{s,0} > m^*_{v,0}$,[2,3,24] and the resulting constraints are shown by the dashed and dotted lines. As expected from the results shown in Fig. 1, effects of nucleon effective mass are small with the value of L shifting by only a few MeV for a given $E_{\rm sym}(\rho_0)$. As one has also expected, effects of varying other macroscopic quantities are even smaller.

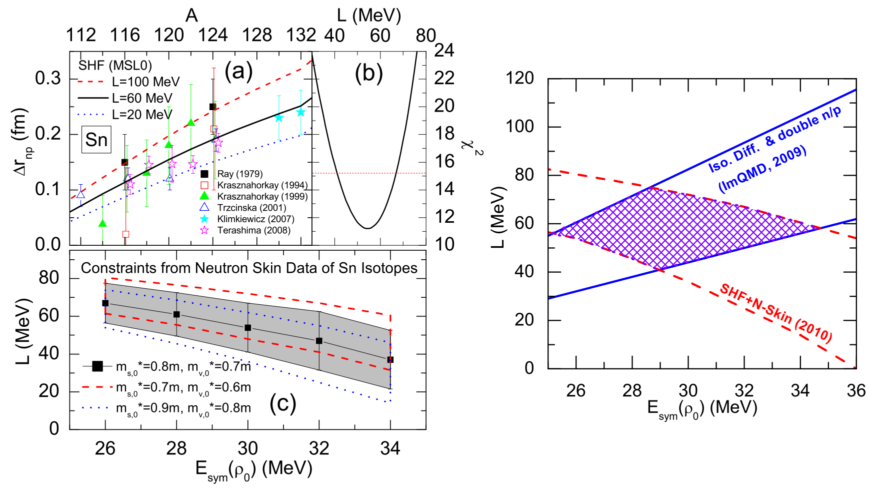

Fig. 2. (Color online) Left window: (a) The Δr_{np} data for Sn isotopes from different experimental methods and results from SHF calculation using MSL0 with $L = 20$, 60 and 100 MeV. (b) χ^2 as a function of L. (c) Constraints on L and $E_{\rm sym}(\rho_0)$ from the χ^2 analysis of the Δr_{np} data on Sn isotopes (Grey band as well as dashed and dotted lines). Right window: Constraints on L and $E_{\rm sym}(\rho_0)$ obtained in the present work (dashed lines) and that from Ref.[25] (solid lines). The shaded region represents their overlap. Taken from Ref.[14]

The above constraints on the L-$E_{\rm sym}(\rho_0)$ correlation can be combined with those from recent analyses of isospin diffusion and double n/p ratio in heavy ion collisions at intermediate energies[25] to determine simultaneously the values of both L and $E_{\rm sym}(\rho_0)$. Shown in the right window of Fig. 2 are the two constraints in the $E_{\rm sym}(\rho_0)$-L plane. Interestingly, these two constraints display opposite L-$E_{\rm sym}(\rho_0)$ correlations. This allows us to extract a value of $L = 58 \pm 18$ MeV approximately independent of the value of $E_{\rm sym}(\rho_0)$. This value of L is quite precise compared to existing estimates in the literature (See Ref.[3] for a recent summary) although the constraint on $E_{\rm sym}(\rho_0)$ is not improved.

4. Summary

We have proposed a new method to analyze the correlations between observables of finite nuclei and some macroscopic properties of nuclear matter, and demonstrated that the existing neutron skin data on Sn isotopes can give important constraints on the symmetry energy parameters L and $E_{\rm sym}(\rho_0)$. Combining these constraints with those from recent analyses of isospin diffusion and double n/p ratio in heavy ion collisions leads to a quite

accurate value of $L = 58 \pm 18$ MeV approximately independent of $E_{\rm sym}(\rho_0)$.

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